IDENTIFICATION OF CRACKS IN BEAMS USING VIBRATIONAL ANALYSIS

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF

Master of Technology
In
Structural Engineering

By

Adyasha Priyadarshini
Roll No. 211CE2237

DEPARTMENT OF CIVIL ENGINEERING

NATIONAL INSTITUTE OF TECHNOLOGY

ROURKELA-769008

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CERTIFICATE

This is to certify that the thesis entitled, ***"IDENTIFICATION OF CRACKS IN BEAMS USING VIBRATIONAL ANALYSIS"*** *submitted by* ***Ms. Adyasha Priyadarshini*** *in partial fulfillment of the requirements for the award of Master of Technology Degree in Civil Engineering with specialization in "Structural Engineering" at National Institute of Technology, Rourkela is an authentic work carried out by her under my supervision and guidance. To the best of my knowledge, the matter embodied in this thesis has not been submitted to any other university/ institute for award of any Degree or Diploma.*

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Adyasha Priyadarshini

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CONTENTS :

ABSTRACT :

Cracks in structural members lead to local changes in their stiffness, flexibility and consequently their static and dynamic behaviour is affected. The influence of cracks on dynamic characteristics like natural frequencies, modes of vibration of structures has been the subject of many investigations. Present work deals with the vibration analysis of an isotropic cantilever beam made with a transverse one-edge non-propagating open crack using the finite element method. The flexibility matrix method is used to calculate the stiffness of the cracked beam here. The vibration of the cracked beam is computed in the present study and then it is compared with the previous study results. The effects of various parameters like crack location, crack depth on the changes in Natural Frequencies of the beam is studied. It is found that, presence of crack in a beam decreases the natural frequency which is more pronounced when the crack is near the fixed support than the free end and the crack depth is more. Then the inverse problem is introduced, as the detection of cracks is very difficult through naked eye and the non-destructive method of detecting cracks which are used is very much costly. Here the first three Natural Frequencies are used to detect the crack depth and location of crack in the beam.

LIST OF SYMBOLS

The principal symbols used in this thesis are presented for easy reference. A symbol is used for different meaning depending on the context and defined in the text as they occur.

Notation	Description
L	Length of the beam
b	Width of the beam
h	Depth of the beam
A	Area of cross-section
a	Depth of the crack
L1	Distance of crack from the fixed end
E	Modulous of Elasticity
I	Moment of Inertia
[M]	Mass Matrix
[K]	Stiffness Matrix
$[\mathbf{K_g}]$	Geometric Stiffness matrix
{q}	Displacement vector
P	External force vector
ω_n	Natural Frequency
ρ	Mass Density
C_{ovl}	Overall Flexibility Matrix
C_{intact}	Flexibility Matrix of an uncracked beam
C_{total}	Total Flexibility Matrix for a cracked beam
K_{crack}	Stiffness Matrix for cracked beam
[L]	TransformationMatrix

CHAPTER 1

1. INTRODUCTION:

What is a Crack?

Crack is a damage that often occurs in members of structures. It is the separation of an object or material into two, or more, pieces under the action of stress**.** Cracks in the any structural systems are very common due to various effects with respect to time, due to natural calamities (such as Earthquake, cyclone; etc.), construction defects, Shrinkage of concrete, chemical reactions in concrete etc.

What are the Effects of Cracks, why it needs to be Analysed?

Generally structures are not collapsed on initial growth of cracks. A crack must be detected in the early state, as it may cause serious failure of the structures with due course of time. However, it is difficult to recognize a crack by visual inspection techniques, when it is too fine. Hence non destructive testing such as vibration technique is used for crack detection. It is essential to study the behaviour of structure having cracks.

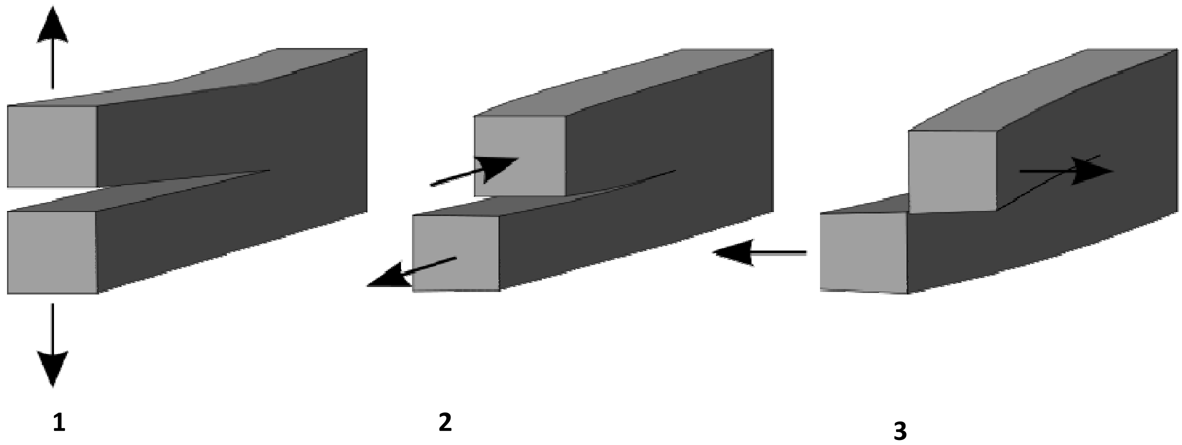
If a structure is defective, there is a change in the stiffness and damping of the structure in the region of the defect. A crack on a structural member introduces a local flexibility which is a function of the crack depth. This flexibility changes the dynamic behavior of the structure, and from this change the crack position and magnitude can be identified.

Major characteristics of structures, which are affected due to presence of crack, are:

• The natural frequency

• The amplitude response due to vibration

• Mode shape.



Cracks are formed due to; 1. Tensile force, 2. In-plane Force and 3. Out-plane Force

CHAPTER 2

2. LITERATURE REVIEW:

Cracks are formed in the structures due to various aspects in due course of time. The formation of cracks does not lead to immediate failure of the structure, it effects gradually. Therefore, the study of crack is required. The cracks sometimes cannot be detected with the naked eye hence non destructive methods of detection of cracks are applied.

The formation of cracks in a structure affects the local stiffness and flexibility of the structure. This problem has been a subject of investigation in many papers, but there are not many papers regarding the Inverse problem method used in this study. In the present study an attempt has been made to the reviews on the isotropic cracked cantilever beam

Kisa et. al(1988) The vibrational characteristics of a cracked Timoshenko beam are analysed. The study integrates the FEM and component mode synthesis. The beam divided into two components related by a flexibility matrix which incorporates the interaction forces. The forces were derived from fracture mechanics expressions as the inverse of the compliance matrix is calculated using stress intensity factors and strain energy release rate expressions.

Gudmundson(1982, 1983) Investigated the influence of small cracks on the natural frequencies of slender structures by a perturbation method as well as by a transfer matrix approach.

Cawley and Adams(1979) have compbined sensitivity analysis and FEM to determine crack location.

Yuen(1985) proposed a systematic finite element approach to determine the relationship between damage location, damage size and corresponding changes in eigen parameters.

Qian et.al Developed a finite element model of an edge cracked beam. They derived the stiffness matrix for a cracked beam by energy method. This stiffness was given two values, one for the closed crack and the other for the open crack.

Rizos et.al. Modeled the crack as a massless rotational spring, whose stiffness was calculated using fractures mechanics. He also conducted experiments to detect crack depth and location from changes in the mode shapes of cantilever beams. A major disadvantage of using mode shape based technique is that obtaining accurate mode shapes involves arduous and meticulous measurement of displacement or acceleration over a large number of points on the structure before and after damage. The accuracy in measurement of mode shapes is highly dependent on the number and distribution of sensors employed.

Lee presented a method to detect a crack in a beam. The crack was not modeled as a massless rotational spring, and the forward problem was solved for the natural frequencies using the boundary element method. The inverse problem was solved iteratively for the crack location and the crack size by the Newton-Raphson method. The present crack identification procedure was applied to the simulation cases which use the experimentally measured natural frequencies as inputs, and the detected crack parameters are in good agreements with the actual ones. The present method enables one to detect a crack in a beam without the help of the massless rotational spring model.

Owolabi et al. used natural frequency as the basic criterion for crack detection in simply supported and fixed-fixed beams. The method suggested has been extended to cantilever beams to check the capability and efficiency. There is need to see if this approach can be used for fixed-free beams.

CHAPTER 3

3. THEORY AND FORMULATION:

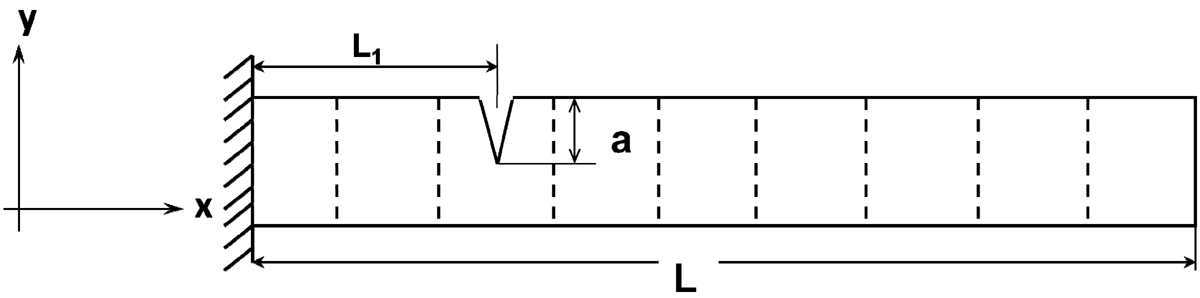


Fig. 1- Schematic diagram of a cracked cantilever beam

The equation of motion in matrix form for vibration of a beam under load is written as

$$[M]\{\ddot{q}\}+[[K]-P[K_g]]\{q\}=0 \tag{1}$$

Where, $[M]$ = Consistent mass matrix

$[K]$ = Bending stiffness matrix of the beam

$[K_g]$ = Geometric stiffness matrix

$\{q\}$ = Displacement vector

P = External force vector

For free vibration the equation (1) can be written as,

$$[M]\{\ddot{q}\}+[K]\{q\}=0 \tag{2}$$

Where, the forcing function, $P=0$

The equation (2) represents an eigen value problem and the roots of the equation give rise to square of the natural frequency given by the equation,

$$[K]-(\omega_n)^2[M]=0 \tag{3}$$

3.1 Elemental Stiffness matrix for Uncracked Beam:

The stiffness matrix for 2 degree of freedom *(v,θ)* for bending in the *xy*-plane for a two-noded Timoshenko beam finite element with shear deformation is line with Gounaris and Papazoglou [1992] as

$$[K] = \frac{EI}{L\left(L^2 + 12\beta\right)}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 + 12\beta & -6L & 2L^2 - 12\beta \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 - 12\beta & -6L & 4L^2 + 12\beta \end{bmatrix}$$

Where, L = length of the element

E = young’s modulus of elasticity

I = moment of inertia of the section with respect to *z*-axis,

and

$$\beta = \frac{EI}{\kappa GA}$$

where, κ = shear correction factor

G = the shear modulus

A = area of the cross - section of the element

For Free Vibration:

α = 0, β = 0

Hence,

$$[K] = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \tag{4}$$

3.2 Elemental Mass matrix for Uncracked beam:

$$\left[M\right] = \int_0^L \left[N\right]^T \left[\rho A\right]\left[N\right] dx \tag{5}$$

$$\left[M\right] = \frac{\rho A l}{420}\begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix} \tag{6}$$

Where, ρ = Mass density of the beam material

A= Cross-sectional area of the beam element

3.3 STIFFNESS MATRIX FOR A CRACKED BEAM ELEMENT

The key problem in using FEM is how to accurately obtain the stiffness matrix for the cracked beam element. The most feasible method is to obtain the total flexibility matrix first and then take inverse of it. The total flexibility matrix of the cracked beam element includes two parts. The first part is original flexibility matrix of the intact beam. The second part is the additional flexibility matrix due to the existence of the crack, which leads to energy release and additional deformation of the structure.

3.3.1 Elements of the overall additional flexibility matrix C_{ovl}

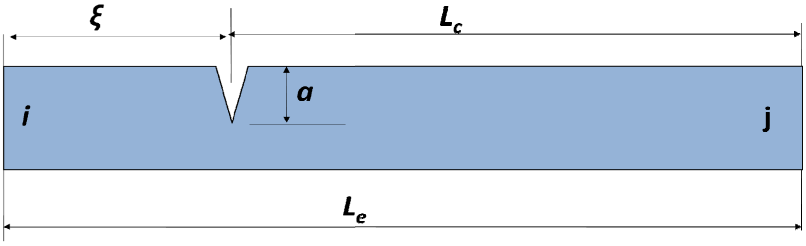


Fig 2 – Diagram of a Typical Beam Element

The above figure 2 shows a typical cracked beam element with a rectangular cross section. The left hand side end node ***i*** is assumed to be fixed, while the right hand side end node ***j*** is free.

b = Breath of the beam

h = Depth of the beam

a = crack depth

L_c= Distance between the right hand side end node **j** and the crack location

Le= Length of the beam element

ξ **=** Distance of the crack form left hand side end node **i**

A = Cross-sectional area of the beam

I= Moment of inertia

According to Dimarogonas *et al.* (1983) and Tada *et al.* (2000) the additional strain energy due to existence of crack can be expressed as

$$\Pi_C = \int_A G dA_C \tag{7}$$

Where, G = the strain energy release rate and

A_C = the effective cracked area.

$$G = \frac{1}{E'}\left[\left(\sum_{n=1}^{2} K_{In}\right)^2 + \left(\sum_{n=1}^{2} K_{IIn}\right)^2 + k\left(\sum_{n=1}^{2} K_{IIIn}\right)^2\right] \tag{8}$$

Where, $E' = E$ for plane stress

$= E/1\text{-}\upsilon^2$ for plane strain

$k = 1 + \upsilon$

K_I, K_{II} *and* K_{III} = stress intensity factors for opening, sliding and tearing type cracks respectively

Neglecting effect of axial force and for open cracks, Eq.8 can be written as

$$G = \frac{I}{E'}\left[\left(K_{I1} + K_{I2}\right)^2 + K_{II1}^2\right] \tag{9}$$

The expressions for stress intensity factors from earlier studies are given by,

$$\left.\begin{aligned} K_{I1} &= \frac{6P_1L_c}{bh^2}\sqrt{\pi\xi}F_1\left(\frac{\xi}{h}\right) \\ K_{I2} &= \frac{6P_2}{bh^2}\sqrt{\pi\xi}F_1\left(\frac{\xi}{h}\right) \\ K_{II1} &= \frac{P_2}{bh}\sqrt{\pi\xi}F_{II}\left(\frac{\xi}{h}\right) \end{aligned}\right] \tag{10}$$

Where,

$$\left.\begin{aligned} F_I(s) &= \sqrt{\frac{\tan\left(\pi s/2\right)}{\left(\pi s/2\right)}}\left[\frac{0.923 + 0.199\left(1 - \sin\left(\pi s/2\right)\right)^4}{\cos\left(\pi s/2\right)}\right] \\ F_{II}(s) &= \frac{1.122 - 0.561s + 0.085s^2 + 0.180s^3}{\sqrt{1-s}} \end{aligned}\right] \tag{11}$$

Here, $s = \dfrac{\xi}{h}$ (crack depth during the process of penetrating from zero to final depth), and $F_1(s)$ *and* $F_{II}(s)$ are the correction factors for stress intensity factors

From definition, the elements of the overall additional flexibility matrix C_{ij} can be expressed as

$$C_{ij} = \frac{\partial\delta_i}{\partial P_j} = \frac{\partial^2 \prod_c}{\partial P_i \partial P_j} \quad (i,j = 1, 2,) \tag{12}$$

Substituting Eq. 10 in Eq. 9 and subsequently in Eq. 7 we get,

$$C_{ij} = \frac{b}{E'}\frac{\partial^2}{\partial P_i \partial P_j}\int\left[\left\{\frac{6P_1L_c}{bh^2}\sqrt{\pi\xi}F_1\left(\frac{\xi}{h}\right)+\frac{6P_2}{bh^2}\sqrt{\pi\xi}F_1\left(\frac{\xi}{h}\right)\right\}^2+\left\{\frac{P_1}{bh}\sqrt{\pi\xi}F_{II}\left(\frac{\xi}{h}\right)\right\}^2\right]d\xi \quad (13)$$

Substituting *i*, *j (1,2)* values, we get

$$C_{11} = \frac{2\pi}{E'b}\left[\frac{36L_c^2}{h^2}\int_0^{\frac{a}{h}} xF_1^2(x)dx + \int_0^{\frac{a}{h}} xF_{II}^2(x)dx\right] \quad (14)$$

$$C_{12} = \frac{72\pi L_c}{E'bh^2}\int_0^{\frac{a}{h}} xF_1^2(x)dx = C_{21} \quad (15)$$

$$C_{22} = \frac{72\pi}{E'bh^2}\int_0^{\frac{a}{h}} xF_1^2(x)dx \quad (16)$$

Now, the overall flexibility matrix C_{ovl} is given by,

$$C_{ovl} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \quad (17)$$

3.3.2 Flexibility matrix C_{intact} of the intact beam element

$$C_{\text{int}\,act} = \begin{bmatrix} \frac{L_e^3}{3EI} & \frac{L_e^2}{2EI} \\ \frac{L_e^2}{2EI} & \frac{L_e}{EI} \end{bmatrix} \quad (18)$$

3.3.3 Total flexibility matrix C_{tot} of the cracked beam element

$C_{total} = C_{intact} + C_{ovl}$

$$C_{total} = \begin{bmatrix} \frac{L_e^3}{3EI}+C_{11} & \frac{L_e^2}{2EI}+C_{12} \\ \frac{L_e^2}{2EI}+C_{21} & \frac{L_e}{EI}+C_{22} \end{bmatrix} \quad (19)$$

3.3.4 Stiffness matrix K_c of a cracked beam element:

From the equilibrium conditions, the stiffness matrix K_c of a cracked beam element can be obtained as

$$K_{crack} = LC_{tot}^{-1}L^T \qquad (20)$$

Where L is the transformation matrix for equilibrium condition

$$L = \begin{bmatrix} -1 & 0 \\ -L_e & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad (21)$$

The results are presented for vibration of beams with cracks using the present formulation. The boundary conditions are

- Fixed end: all degree of freedoms are constrained
- Free end: no restraint

CHAPTER 4

4. RESULTS AND DISCUSSION:

The effect of an open edge transverse crack on various parameters of a beam like vibration is studied and is compared with previously studied results. The formulation is then validated and extended for other problems.

4.1 Introduction:

In order to check the accuracy of the present analysis, the experiment done by Kisa et al (1988) was performed again, the case is considered to validate the program. The method described has been applied to a transversely cracked Timoshenko beam.

The properties of the material are given below:

Breath of the beam = 0.025 m

Depth of the beam = 0.0078 m

Length of the beam = 0.2 m

Elastic modulus of the beam = 216×10^9 N/m^2

Poisson's Ratio = 0.28

Unit Weight = 7.85x103 kg/m^3

End condition of the beam = One end fixed and other end free (Cantilever beam).

4.2 CONVERGENCE STUDY:

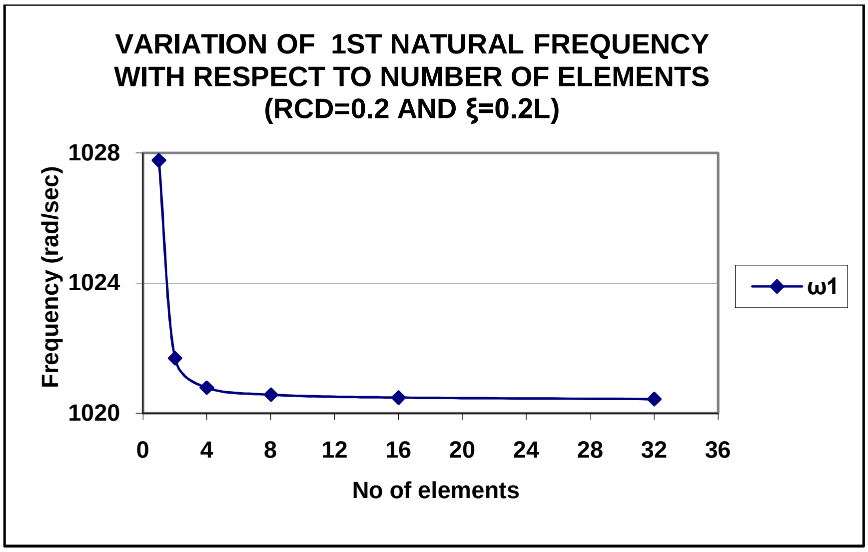


Fig 3- Convergency study of the 1st Natural Frequency

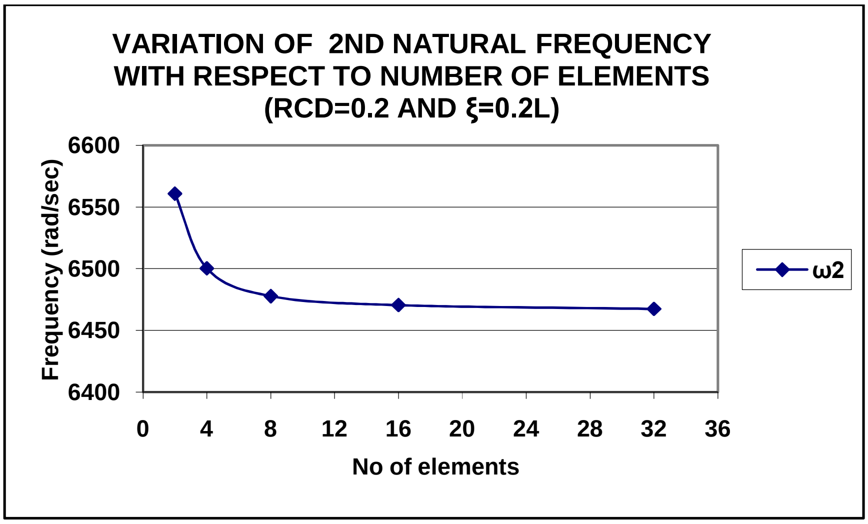


Fig 4- Convergence study of 2nd Natural Frequency

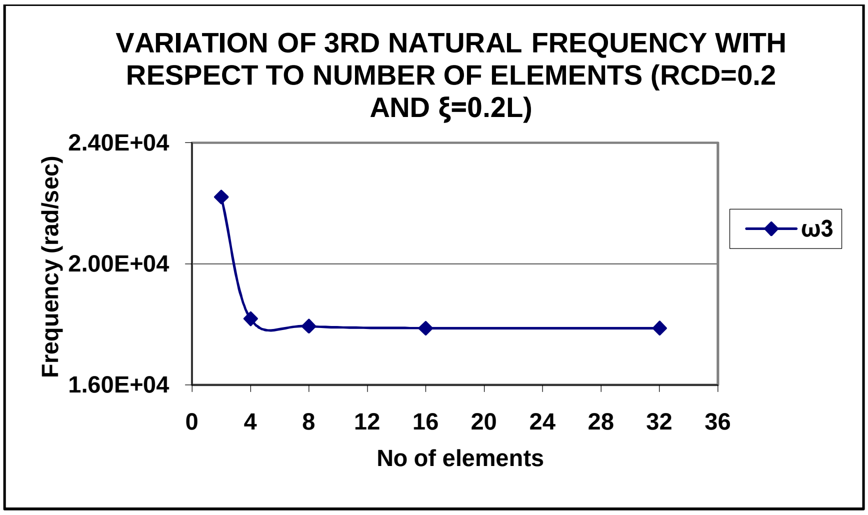


Fig 5 - Convergence study of 3rd Natural Frequency

4.3 Validation of the Program:

The method and the case considered by Kisa *et al.[3]* is used here, a Fortran code was generated according to the case. Here the code is validated by comparing the results of the present analysis that was done with the results that was obtained earlier.

Table- I Showing the Comparative study in between Kisa et.al. and Present Analysis

Crack position (*L1*)	**Relative crack depth (*a/h*)**	**Mode I**		**Mode II**		**Mode III**	
		Present analysis	**Kisa *et al.*[3]**	**Present analysis**	**Kisa *et al.*[3]**	**Present analysis**	**Kisa *et al.*[3]**
Intact	**0**	**1037.276**	**1037.019**	**6465.828**	**6458.343**	**17953.056**	**17960.564**
x = 0.2L	**0.2**	**1020.476**	**1020.137**	**6470.531**	**6457.396**	**17880.233**	**17844.860**
	0.4	**967.221**	**966.952**	**6465.811**	**6454.483**	**17608.277**	**17596.570**
	0.6	**842.353**	**842.220**	**6451.411**	**6448.175**	**17006.400**	**16944.560**
	0.8	**551.278**	**551.046**	**6397.712**	**6436.008**	**15873.352**	**15512.550**
x = 0.8L	**0.2**	**1040.107**	**1036.884**	**6465.478**	**6440.057**	**17954.738**	**17758.610**
	0.4	**1039.154**	**1036.414**	**6403.877**	**6375.921**	**17297.240**	**17077.990**
	0.6	**1037.516**	**1034.943**	**6211.675**	**6174.710**	**15589.514**	**15286.830**
	0.8	**1028.324**	**1026.769**	**5252.677**	**5169.264**	**11769.558**	**11353.180**

4.4 Example Problem:

An aluminium cantilever beam, having the following properties was taken into consideration and its Natural Frequency was noted for uncracked section and cracked section.

Dimensions:

L=0.3m, b=0.01m, H=0.01m

Properties:

E=69.79Gpa, $\rho=2600kg/m^3$ and υ=0.28

No. of Elements=16

L_e = 0.01875

Assumptions:

- The Analysis is linear. This implies both linear constitutive relations (generalized Hooke’s law for the material and linear kinematics).
- The displacement is small to accommodate small deformation theory.
- Damping is neglected

After obtaining the comparison with the previous study and present validating the formulation, the FORTRAN coding was used to find out the theoretical first three Natural Frequencies of the considered beam, then graphs were plotted for the obtained results.

4.4.1 RELATIVE POSITION OF CRACK v/s FREQUENCY GRAPH

Table II- 1st Numerical Natural Frequency at different crack depth amd crack position

1st FREQUENCY					
L1/L	**RCD=0.0**	**RCD=0.1**	**RCD=0.3**	**RCD=0.5**	**RCD=0.7**
0.10	583.888	580.763	557.524	502.695	383.864
0.30	583.888	582.491	571.562	542.776	462.861
0.50	583.888	583.443	579.751	569.256	533.387
0.70	583.888	583.826	583.221	581.450	574.752
0.90	583.888	583.889	583.879	583.850	583.739

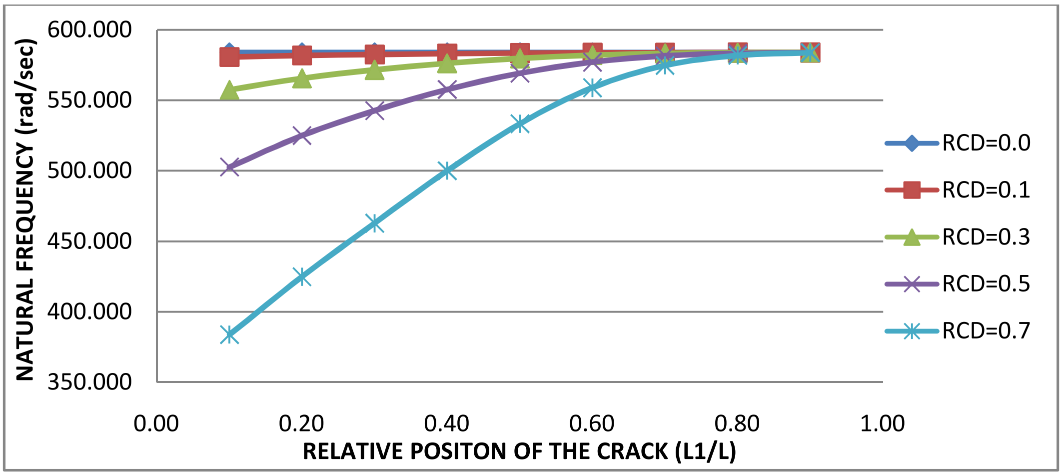


Fig 6 – 1st Natural Frequency.

From the above figure we can see that the natural frequency of the beam decreases with increase in the crack depth, whereas it is also observed that when the position of the crack moves from the fixed end towards the free end of the cantilever beam, the effect of the crack also decreases gradually.

Table III- 2nd Numerical Natural Frequency at different crack depth and crack positon

2nd FREQUENCY					
L1/L	**RCD=0.0**	**RCD=0.1**	**RCD=0.3**	**RCD=0.5**	**RCD=0.7**
0.10	3644.322	3638.814	3579.215	3457.485	3265.385
0.30	3644.322	3643.431	3623.887	3574.497	3453.721
0.50	3644.322	3631.198	3533.891	3298.029	2767.967
0.70	3644.322	3637.907	3583.365	3432.837	2977.262
0.90	3644.322	3644.438	3642.636	3637.204	3616.035

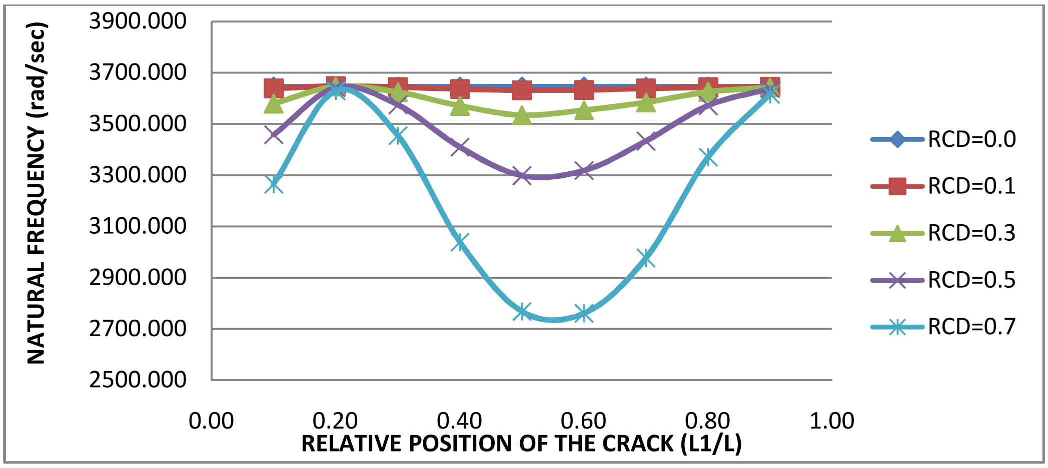


Fig 7- 2nd Natural Frequency

From the above figure it was found out that the 2nd natural frequency is not affected at 20% length of the beam and at the extreme free end, whereas the crack affects the natural frequency the beam to the maximum limit at 50 – 60% of the length of the cantilever beam.

Table IV- 3rd Numerical Natural Frequency at different crack depth and crack position

3rd FREQUENCY					
L1/L	**RCD=0.0**	**RCD=0.1**	**RCD=0.3**	**RCD=0.5**	**RCD=0.7**
0.10	10139.102	10152.481	10102.917	10000.484	9836.839
0.30	10139.102	10109.591	9892.597	9406.876	8507.232
0.50	10139.102	10150.281	10149.040	10133.841	10072.062
0.70	10139.102	10099.120	9808.434	9152.874	7946.929
0.90	10139.102	10140.279	10110.678	10019.608	9640.155

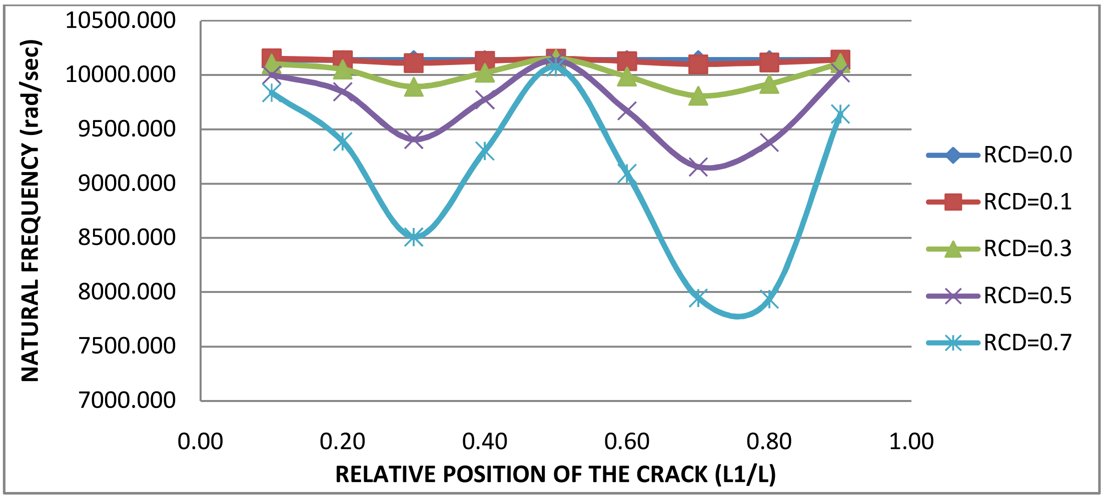


Fig 8 - 3rd Natural Frequency

From the above figure it was observed that the 3rd natural frequency of the beam is not affected at the fixed end, free end and at the middle of the beam, whereas the frequency is very much reduced at 30% and 80% of the length of the beam.

4.4.2 The Experimental setup:

After the getting the theoretical values by the programs, the values were cross checked by doing experiment using FFT, to check how the values of the natural frequency can vary experimentally.

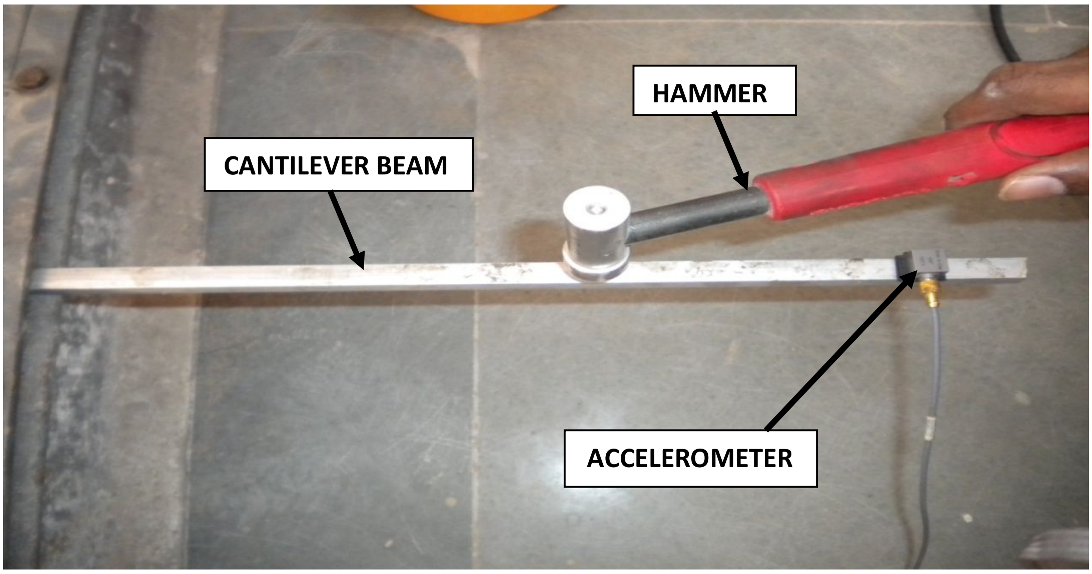


Fig 9 – Experimental setup of FFT analyser.

In the experimental setup there is an aluminium cantilever beam of cross section b=h=0.01m and length L=0.3m. An accelerometer is attached to the beam to note down the natural frequency of the beam. An impact hammer is used to induce vibration to the beam.

The beam is struck lightly with the impact hammer and natural frequencies of the beam is recorded FFT in the computer and noted down.

4.4.3 FFT Output:

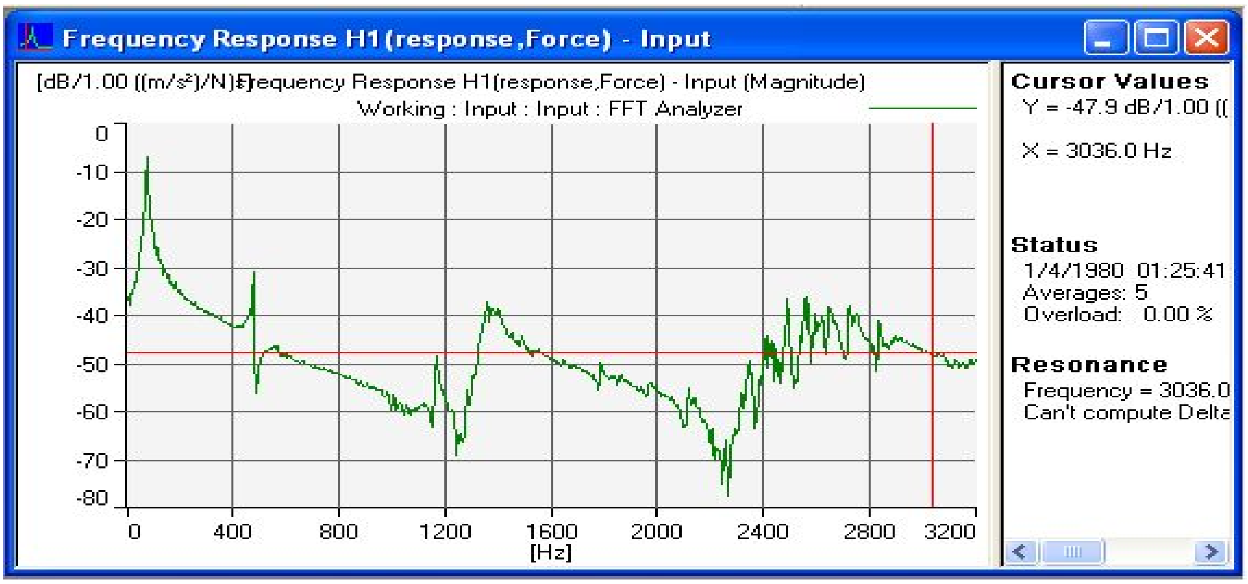


Fig 10 – Output Graph of FFT analyser.

Frequency of the Beams Obtained from FFT :

The values obtained in the experiment was almost equal to values of the theoretical analysis, the variations in the natural frequencies were also same.

Table no. V

Experimental Natural Frequency Of an Uncracked Beam:		
f 1	f 2	f 3
92	579	1613
91	580	1612
90	581	1610
92	580	1612

Table no. VI

Experimental Natural Frequency Of a Cracked Beam:		
RCD = 0.5 Crack Position = 3cm		
f 1	f 2	f 3
79	550	1591
78	552	1590
80	551	1591
79	550	1589

Table no. VII

Experimental Natural Frequency Of a Cracked Beam:		
RCD = 0.3 Crack Position = 3cm		
f 1	f 2	f 3
88	569	1607
89	568	1608
88	569	1607
87	570	1609

Table no. VIII

Experimental Natural Frequency Of a Cracked Beam:		
RCD = 0.3 Crack Position = 15cm		
f 1	f 2	f 3
92	560	1615
90	557	1615
92	565	1614
91	554	1614

Table no. IX

Experimental Natural Frequency Of a Cracked Beam:		
RCD = 0.5 Crack Position = 15cm		
f 1	f 2	f 3
88	524	1615
90	524	1615
90	523	1614
89	522	1614

4.5 Inverse Problem:

In the inverse problem, here the natural frequencies of the beam is used to find out the crack position or location and depth of the crack.

The procedure for the Inverse problem is:

1. The first three natural frequencies of the beam is measured.
2. The measured frequencies are normalised.
3. The contour lines from different modes are plotted on the same axes, in a 3D graph (Normalised Frequencies, Crack location, Crack depth), and
4. The Location of the point(s) of intersection of the different contour lines. The point(s) of intersection, common to all the three modes, indicate(s) the crack location, and crack depth.

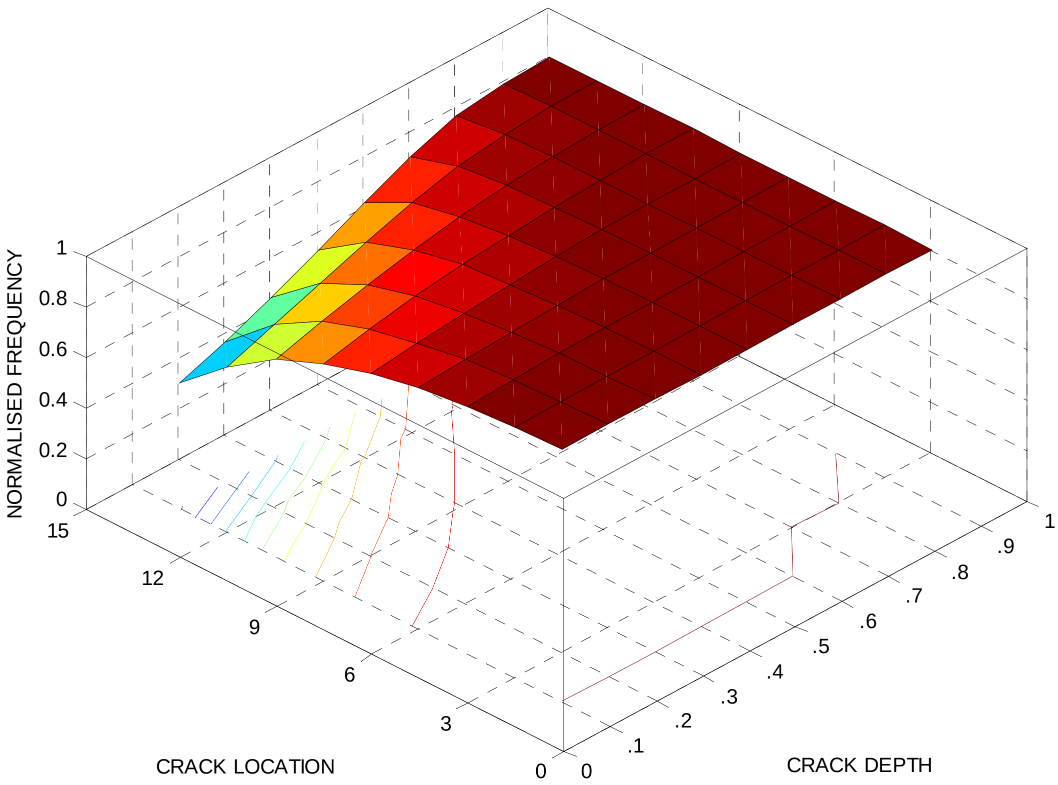


Fig 11 – 3D Graph plot of the 1st normalized natural frequency, crack location and crack depth with contouring.

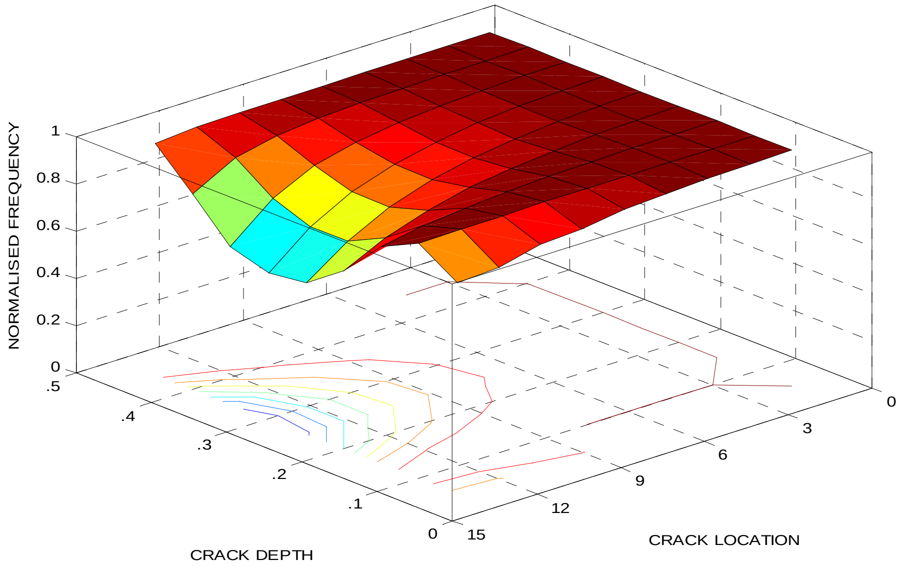


Fig 12 – 3D Graph plot of the 2nd normalized natural frequency, crack location and crack depth with contouring.

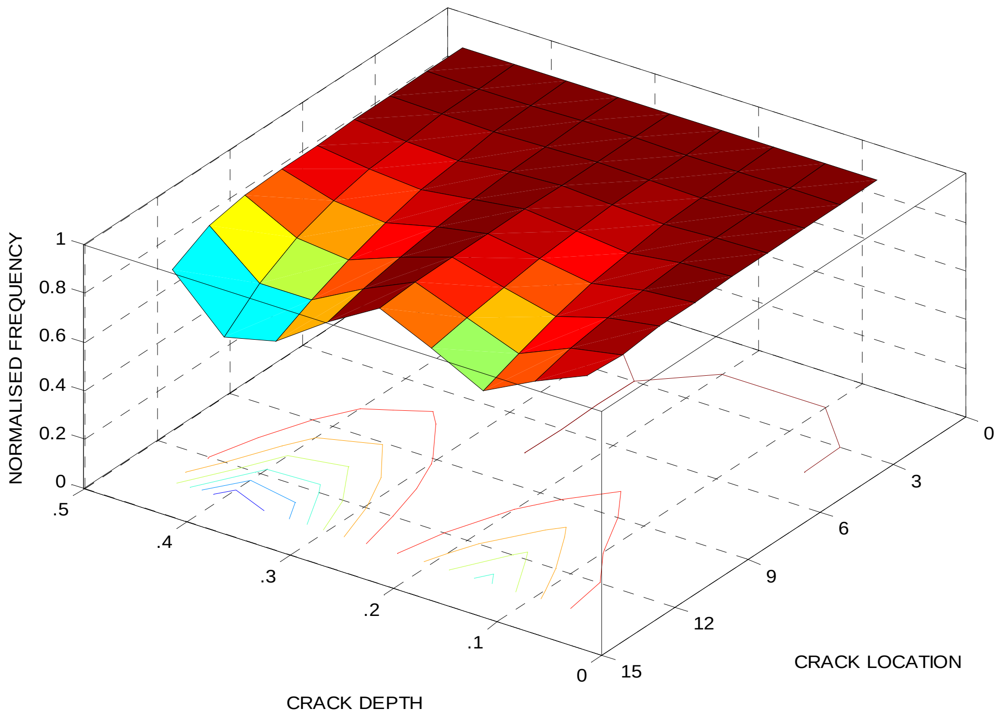


Fig 13 – 3D Graph plot of the 3rd normalized natural frequency, crack location and crack depth with contouring.

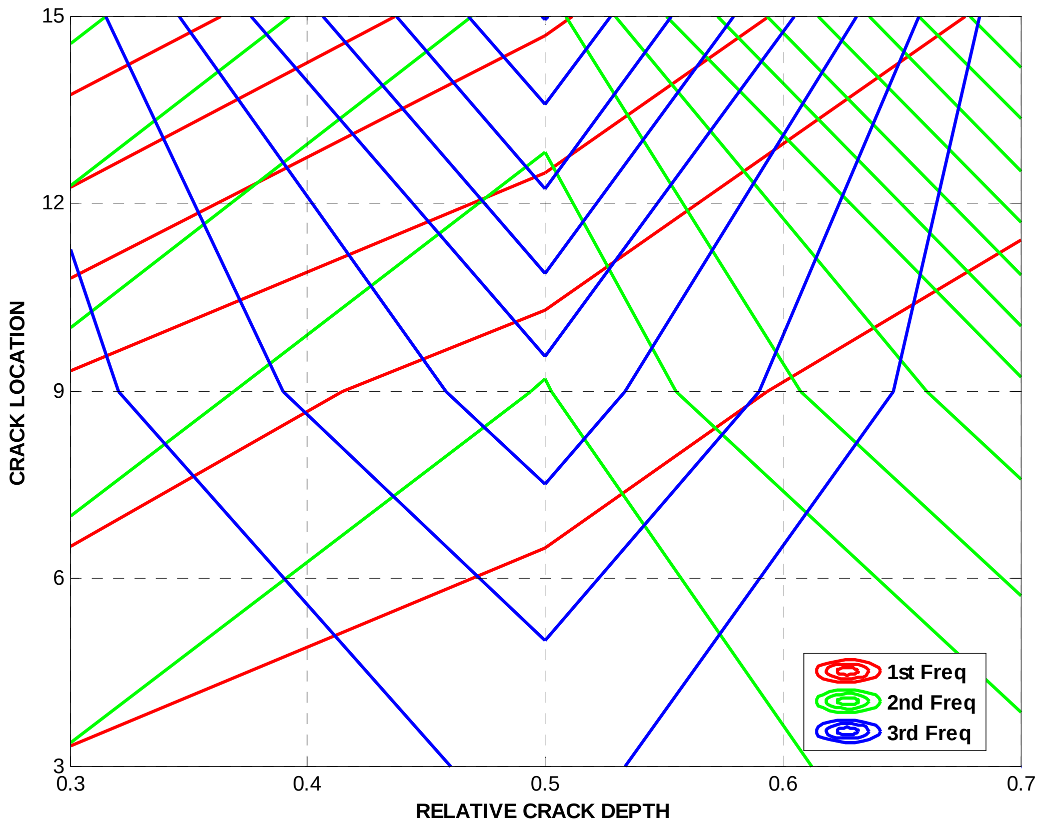


Fig 14 – Contour graph of 1st three normalized natural frequency showing RCD=0.7 and crack location= 15 cm, RCD= 0.5 and crack location 12cm.

From the fig 14. We can see point where 1st three natural frequency of the beam coincides with each other that particular point shows the exact crack location and crack depth in the beam.

CONCLUSION:

- The frequency of the cracked cantilever beam decreases with increase in the crack depth for the all modes of vibration.
- When the crack location shifts towards the fixed end of the cantilever beam the natural frequency decreases in first mode of vibration.
- But for second, third and fourth modes of vibrations the frequency of the cracked beams for the same crack depth varies as sinusoidal (approx).
- The effect of crack is more near the fixed end than at far free end.
- The exact approx location of the crack and the depth can be computed by the natural frequencies of the beam (1st three natural frequencies are taken into consideration).

CHAPTER 5

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